Coalition and Victory, The Correlated Equilibria of Shareholders Costly Voting Game

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Preliminary Version, Comments Welcome

December 7, 2008

Abstract

This paper investigates the existence of correlated equilibria of a shareholders' costly voting game. It is shown that there exists a correlated equilibria component where a minimum winning coalition of the supporters of status quo and a dominant shareholder vote with strictly positive probabilities and all other players abstain. This result is different from the previous one suggested by Ritzberger (2005) where the solution concept was pure Nash equilibrium and only dominant shareholder participates in the assembly. This new result suggests a different behavior for the manager of firm in equilibrium.

1 Introduction

The study of outcomes of shareholders voting games is interesting because from a practical point of view it is important to calculate the value of voting rights of a particular shareholder. The literature of corporate finance has reviewed the issue of control rights extensively (see for example Jensen and Ruback (1983), Shleifer and Vishny (1986)) and has used it to rationalize investment decisions for instance in mergers and acquisitions. The voting power of every single share

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(the intangible commodity traded in the market for corporate control) depends mainly on the possibility of influencing corporate key decisions. The extend a shareholder can be decisive varies crucially with her stake in the firm, the distribution of other shares, decision making rules and the incentive of other shareholders to cast their vote or abstain. As a result, game theoretical models are the natural device to study such environment. Unfortunately, there exist only few studies who use non-cooperative game theory to look at the behavior of shareholders in the context of corporate democracy and this opens the stage for further research.

The question investigated here is the properties of equilibria of a costly shareholder voting game when the signals players receive are correlated with each other. Correlation of signals means that in real world the players of a simultaneous-move game do not choose their strategies in the vein of ignorance (and just flip a coin in the case of a mix strategy) but can have updated beliefs about the strategies of others when choosing a particular strategy. To get to such correlation in practice, one can think of any sort of an implicit "correlating device" which can be for instance the common memories from previous experiences or columns in business press. To model the situation, I use the concept of correlated equilibria *'a la* Aumann (Aumann, 1987). Correlated equilibrium refers to a set of recommendations player receive from an umpire. They do not observe or receive signals about the strategies of others but do have correct (I tend to say in a rational expectations sense) information about the probability that any outcome realizes. A recommendation turns out to be a correlated equilibrium of the game if and only if each player finds it optimal to follow that recommendation taking it as granted that the others do so.

A very closely related paper to the current one is Ritzberger (2005) which shows that the pure Nash equilibrium of the game exists iff there exists a dominant shareholder who votes for the alternative and commands more shares than any other shareholder does. Under this equilibrium only the dominant shareholder votes and everybody else finds it optimal to abstain. This strong result implies that no matter how large the share of the other party is, the manager should always follow the desires of dominant shareholder. I show in this paper that the results might be different (at least in some cases) if one considers the correlated equilibrium of the game.

The reader should observe that the cost of voting is also an essential element in our analysis. If participation is not costly, then voting is always better than abstain as long as there is no information asymmetry among shareholders. Information asymmetry concerns is valid since Feddersen and Pesendorfer (1996) in their seminal paper titled "'the curse of swing voter"' explain why an uninformed shareholder may find it optimal to trust the other shareholders and not vote even if voting is costless. I abstract from this aspect and assume that every player clearly knows her preference, her type and the pay-off of each outcome in order to focus on the "'cost of voting"' aspect of the problem.

Costly voting is a plausible and realistic assumption. Imagine a diversified shareholder who owns stakes in several companies whose HQs are located in different cities. In order to vote, she either needs to travel there bearing all time and monetary costs or use a proxy. Even using a proxy is not costless because a careful shareholder would like to study company performance and the proposals on the table, before delegating her rights to a proxy and this again requires times and resources.

This paper goes as follow. First, a short summary of the relevant literature is provided. Then, the formal model of the game and the equilibrium are presented. To fix the idea an illustrating example is used in the next section. Finally, a general existence proof for the correlated equilibria is given which is followed by arguments showing that the correlated equilibria components differs essentially from the Nash results. This is important to prove since the Nash equilibrium of every game contains some correlated equilibria and therefore only the correlated equilibria outside of the convex hull of Nash equilibrium are interesting.

2 Related Literature

The need to implement a voting mechanism inside of a corporation arises from the fact that markets are not complete (e.g. Grossman and Hart, 1979) or the competition is not perfect (e.g. Dierker, 2003). Without market completeness, the price of Arrow securities (or state-contingent claims) is not unique and therefore the present value of different production decisions can not determined uniquely in the market. Moreover, if the firm enjoys some form of market power then those shareholders who are at the same time the consumers of the firm, may have different preference over production plans. Under these assumptions, the text-book version conception of firm's objectives is ambiguous and not a well-defined concept. Indeed it is more plausible to consider firm's objectives from the perspective of each shareholder rather than talking of the objectives of firm as an atomistic entity. It is an old agreement in the arena of political science that because the problem of preference aggregation exists, decisions regarding the provision of public goods should be handled through some form of social choice mechanisms (for instance elections in democratic systems). Likewise, the failure of shareholders unanimity motivates the need to implement similar mechanisms inside of a firm. One globally used form of this corporate social choice mechanism is shareholders assembly and the majority rule.

The political democracy, however, faces a paradox. Considering that voting is costly (one has to dress up and walk to the voting places) and there are Thousands or Millions of other people who usually vote, the probability of being pivotal is extremely small for each citizen. Indeed each person is pivotal if and only if exactly equal number of people vote for two candidates and her vote then turns the results into the victory of her favorite party. One can see that this event realizes with a very tiny probability. If the expected benefits of taking an action (voting here) is so small even after comparing with small costs of voting, then a valid question would be why people vote? Apart from all sentiment and sense of civic responsibility arguments, one can also build a rational choice model for such story.

Palfrey and Rosenthal (1993) paper is one successful attempt to use a game-theoretical approach to model and explain this paradox. They model voting as a simultaneous-move game and show that there exist mixed strategy equilibria where a significant portion of players participate and vote despite the fact that the probability of decisiveness for each player is very small.

This result is however less relevant for corporations because according to various corporate chapters around the world, each shareholder's voting power is usually a non-decreasing function of her shares. As a result, corporate democracy differs from the political one in two very important aspects. First, the voting power of different voters is different (whereas political democracy usually functions based on the so-called one-head-one-vote principle) and the second difference is that the shareholder votes can basically be traded in the market (although there might be legal barriers for that).

There is another line of literature looking to the problem from a cooperative game theory perspective. This approach considers all possible coalitions among shareholders and then allocates voting power indexes according to these coalitions. Shapley-Shubik index and Bazhnaf index are the two most famous ones which basically calculate the number of occasions where every player is pivotal. Denis (1987, 2003) shows how to compute this index and also mentions that the power index has a very strange non-linear and even non-monotone relation with the ratio of shares between different players. The problem with cooperative solution is that it requires strong form of coordination and enforcement mechanims which are rare in reality. I therefore use correlated equilibria which is a semi-cooperative concept but is a self-enforcing equilibrium concept. However, in this paper I will benefit from some modeling ideas presented in cooperative game theory literature.

3 The Model

The game consists of :
1) The set of N≥2 shareholders denoted by I.
2) The set of pure strategies s=$\{0, 1\}$
3) The Cartesian product of individual strategies defining outcome space T= $\{0, 1\}^n$
4) The pay-off U_i which is a function of the result and the small cost of voting ϵ
5) A probability distribution ϕ over the outcomes space

There exist a subset of shareholders who intend to challenge current policies. Therefore, the set of players is partitioned into two subsets A and B that support the status-quo and the change respectively. A positive number of members belong to each subset and A ∪ B = I , A ∩ B=ϕ, meaning that no shareholder is in favor of both options. Tie breaking rules support the status-quo and says that if no shareholder shows up in the voting assembly, current policies will continue. There exits a dominant shareholder denoted by D hereafter. The voting shares are denoted by θ_j and the existence of dominant shareholder means ∃ D s.t. $\theta_D \geq \theta_i$, ∀ i ∈ I. Voting involves a small participation cost $\frac{\epsilon}{\Theta}$ that is strictly positive, normalized to the number of shares for all players and independent of the outcomes. Since participating and voting against one's desired alternative is strictly dominated by abstaining, the pure strategy space of each player consists of only two admissible strategies $\{0, 1\}$ where 0 means not voting and 1 refers to voting. The strategy space of the game is therefore the product space of the pure strategy sets and we show every outcome (pure strategy combination) by a n-vector. The pay-off of winning the game is normalized to 1 for each share independent of the player who owns it. The pay-off to each player is a function of her own strategy, the number of shares she holds and the state of the world (outcome of the game) which is either Win or Not Win. As a result, the pay-offs are as follow: u_i(1,win)=1-$\frac{\epsilon}{\Theta}$, u_i(0,win)=1 , u_i(1,not win)=-$\frac{\epsilon}{\Theta}$, u_i(0,not win)=0. This formulation takes into account the fixed cost of voting (assumed an equal opportunity cost for each shareholder going to the assembly) which should be normalized by the number of shares the person holds, hence a more costly action for the small shareholders. It is also beneficial to point that the pay-off presented here demonstrates a lexicographical type of preference where the shareholder prefers wining without voting to voting and wining and so on.

3.1 Correlated Equilibria

To introduce the correlated equilibria of the game one needs to describe the vector space S of outcomes of the game. As said before every player chooses her strategies from the set $\{0, 1\}$ and the strategies of all players together constitute a vector S. The vector space which contains all such vectors of S is T.

Every correlated equilibrium of a game is a probability distribution φ over T (the space of all possible outcomes) such that:

$$\sum_{S_{-i} \in T_{-i}} \varphi(S_{-i}, S_i)(u_i(S_{-i}, S_i) - u_i(S_{-i}, S_i^{'})) \geq 0, \forall S, S^{'} \in T_i$$

φ is a function that maps every element S of T into the unit interval and sums up to one i.e. φ : T $\longrightarrow$ [0,1] , $\sum_{S \in T} \varphi(S) = 1$.

Since the number of players is finite, the domain of φ (outcome space) is finite as well and one can represent φ by a vector of dimension 2^n. Each element of this vector (i.e. φ_i) represents the probability of realization of an specific outcome S. Each member of T (i.e. S $\in$ T) is itself a n-vector which takes values from the set s=$\{0, 1\}$.

To make the definition more clear, I use an example of a simple 2-by-2 matrix game. Suppose each player can play either L or R. In this example s=$\{L, R\}$, T=$\{(L, L),(L, R),(R, R),(R, L)\}$. Finally, ϕ will assign probabilities to each of these four outcomes and the sum of these probabilities should be one. For instance ϕ might be:

	L	R
L	0.2	0.4
R	0.1	0.3

Which means that the outcome (L,L) occurs with probability 0.2, etc. Notice that a big difference of correlated equilibria and Nash equilibrium is that the latter is defined on independent marginal probabilities over individual strategies while the former considers "joint" probabilities. To see it more clearly, try to find mix strategies which generate the previous outcome probabilities. Do not waste your time, there does not exist such marginal probabilities! Finally, notice that ϕ only defines a probability over outcomes whether a particular probability distribution is a correlated equilibrium or not depends on if that probability distribution satisfies equilibrium condition.

Coming back to the problem, the space of pure strategies consists of only two strategies therefore there exist two inequalities associated with each player:

$$\sum_{S_{-i} \in T_{-i}} \varphi(S_{-i}, 1)(u_i(S_{-i}, 1) - u_i(S_{-i}, 0)) \geq 0$$

$$\sum_{S_{-i} \in T_{-i}} \varphi(S_{-i}, 0)(u_i(S_{-i}, 0) - u_i(S_{-i}, 1)) \geq 0$$

At this stage the key concept of pivot (or decisiveness) is introduced. In the theory of voting being pivotal means that the player has the ability to change the result of voting. Formally, the set of pivot opportunities for each player i includes all pure strategy combinations of other players that if i switches from 0 to 1 the result of the game would change from not winning her desired result to winning it and vise versa. Denote by W_i the set of all pure strategy combinations of other players where the player i is pivotal. Moreover let G be the event that player i's desired outcome emerges. Then $W_i = \{S_{-i} \in T_i$: P(G $|(1, S_{-i})$)=1 and P(G $'|(0, S_{-i})) = 0\}$

For each player, the space of outcomes can be portioned into two subsets:
1) All outcomes where the player is pivotal
2) The outcomes where she is not pivotal.
Moreover, the pay-off structure of the game allows us to summarize the pay-off difference between playing 1 and 0. Two different cases can be distinguished:

1) The player is pivotal (i.e. $S_{-i} \in W_i$): In this case $u_i(S_{-i}, 1) - u_i(S_{-i}, 0)$ $= 1 - \frac{\epsilon}{\Theta_i}$

2) The player is not pivotal (i.e. $s_{-i} \notin W_i$): $u_i(S_{-i}, 1) - u_i(S_{-i}, 0) =$ - $\frac{\epsilon}{\Theta_i}$

These results are very intuitive. If the player is pivotal then participation will impose a small fix cost of $\frac{\epsilon}{\Theta_i}$ while she will gain a pay-off 1 out of changing the result toward her desired outcome. As a result, the total pay-off gain is 1-$\frac{\epsilon}{\Theta_i}$. On the other hand, if she is not pivotal and switches from abstain to vote she has to incur an extra cost of $\frac{\epsilon}{\Theta_i}$ but receives no further gain.

Consider the first inequality and observe that using the idea of partitioning the outcome space for each player to pivot and non-pivot outcomes, one can split $\sum_{S_{-i} \in T_{-i}} \varphi(S_{-i}, 1)(u_i(S_{-i}, 1) - u_i(S_{-i}, 0))$ into two components:

$$\sum_{S_{-i} \in W_i} \varphi(S_{-i}, 1)(u_i(S_{-i}, 1) - u_i(S_{-i}, 0)) + \sum_{S_{-i} \notin W_i} \varphi(S_{-i}, 1)(u_i(S_{-i}, 1) - u_i(S_{-i}, 1)) \geq 0$$

To save the notation I define :
P(pivot | Vote) $= \sum_{S_{-i} \in W_i} \varphi(S_{-i}, 1)$ and P(pivot | Abstain)$=\sum_{S_{-i} \in W_i} \varphi(S_{-i}, 0)$.
Now plug-in the previous results for $u_i(s_{-i}, 1) - u_i(s_{-i}, 0)$ into this equation:

$\sum_{S_{-i} \in W_i} \varphi(S_{-i}, 1)(1 - \frac{\epsilon}{\Theta_i}) + \sum_{S_{-i} \notin W_i} \varphi(S_{-i}, 1)(-\frac{\epsilon}{\Theta_i}) =$
$\sum_{S_{-i} \in W_i} \varphi(S_{-i}, 1) + \sum_{S_{-i} \in W_i} \varphi(S_{-i}, 1)(-\frac{\epsilon}{\Theta_i}) + \sum_{S_{-i} \notin W_i} \varphi(S_{-i}, 1)(-\frac{\epsilon}{\Theta_i}) =$
$\sum_{S_{-i} \in W_i} \varphi(S_{-i}, 1) + \sum_{S_{-i} \in T_{-i}} \varphi(S_{-i}, 1)(-\frac{\epsilon}{\Theta_i}) \geq 0 \Longrightarrow \theta_i \sum_{S_{-i} \in W_i} \varphi(S_{-i}, 1) \geq$
$\frac{\epsilon}{\Theta_i} \sum_{S_{-i} \in T_{-i}} \varphi(S_{-i}, 1) \Longrightarrow P(pivot|$ voting$) \geq \frac{\epsilon}{\Theta_i} \sum_{S_{-i} \in T_{-i}} \varphi(S_{-i}, T)$

The same argument applies for the second inequality with only difference that the direction of inequality will be reversed. Therefore the system of inequalities for each player eventually boils down into two individual rationality conditions:

P(pivot | voting) $\geq \frac{\epsilon}{\theta_i} \sum_{S_{-i} \in T_{-i}} \varphi(S_{-i}, 1)$ (1)

P(pivot | abstain) $\leq \frac{\epsilon}{\theta_i} \sum_{S_{-i} \in T_{-i}} \varphi(S_{-i}, 0)$ (2)

Notice that if one could decompose the probability distributions φ_i into the product of some marginal distributions (the case of Nash equilibrium) it was possible to calculate $\sum_{s_{-i} \in S_{-i}} \varphi(s_{-i}, 1)$ and $\sum_{S_{-i} \in T_{-i}} \varphi(s_{-i}, 0)$ using marginal distributions and get into a different set of inequalities:

σ_i P_i(Pivot) $\geq \sigma_i \frac{\epsilon}{\Theta_i}$ (3)

(1-σ_i) P_i(Pivot) $\leq$ (1-σ_i)$\frac{\epsilon}{\Theta_i}$ (4)

Obviously, the first set (correlated equilibria) provides much larger solution space than the second one (Nash equilibrium), because allowing the probability distributions to be written as the product of marginal reduces the dimension of solution space substantially (indeed from 2^n-1 to only n(2-1)=n). Both sets will be used later to compare the correlated equilibria with the convex hull of solutions coming out of the Nash equilibrium setting.

One further remark is that if players play with fully mixed strategies (i.e. σ_i $\in$ (0,1)) it is legitimate to drop σ_i from both sides of inequalities (3) and (4) get one equality instead of two as P(pivot)= $\frac{\epsilon}{\Theta_i}$. This result is also intuitive. When P(pivot)= $\frac{\epsilon}{\Theta_i}$ the player is indifferent between abstaining (getting nothing) and voting and again getting nothing , since the expected pay-off of voting is $\frac{\epsilon}{\Theta_i}$ * $(1 - \frac{\epsilon}{\Theta_i})$ + $(1 - \frac{\epsilon}{\Theta_i})$ * (-$\frac{\epsilon}{\Theta_i}$) = 0. On the other hand, with pure strategies (i.e. s $\in \{0, 1\}$) one of the inequalities disappears and the other one remains as P(Pivot) $\geq \frac{\epsilon}{\Theta_i}$ (P(Pivot) $\leq \frac{\epsilon}{\Theta_i}$ resp) when S=1 (S=0 resp).

4 An Illustrating Example

Consider a three-player game with the following setting:
A= $\{1,2\}$ and B=$\{3\}$. $\theta_1 = \theta_2 = 0.3, \theta_3 = 0.4, \epsilon = 0.05$. The Nash equilibrium of the game consists of three elements:

1) Pure Nash: S1=(0,0,1)

2) Fully Mixed: $\sigma1$= ($\sqrt{1-\frac{\epsilon}{0.4}}$, $\sqrt{1-\frac{\epsilon}{0.4}}$, $\frac{\epsilon}{0.3\sqrt{1-\frac{\epsilon}{0.4}}}$)

3) Pure and Mixed: $\sigma2$ = ($\frac{\epsilon}{0.3}$, $\frac{\epsilon}{0.3}$,1)

Now, we show that there exits "at least" one component of correlated equilibria outside the convex hull of Nash. Representing the outcomes by a vector of 3, label the probability distributions as follow: φ_1=$\varphi(1,1,1)$, φ_2=$\varphi(1,0,1)$, φ_3=$\varphi(0,1,1)$, φ_4=$\varphi(0,0,1)$, φ_5=$\varphi(1,1,0)$, φ_6=$\varphi(1,0,0)$, φ_7=$\varphi(0,1,0)$, φ_8=$\varphi(0,0,0)$

The pivot set for each player will be then as follow:

1) Player 1: $\{(1,1,1),(0,1,1)\}$
2) Player 2: $\{(1,1,1),(1,0,1)\}$
3) Player 3: $\{(0,0,1),(1,0,1),(0,1,1),(0,0,0),(0,1,0),(1,0,0)\}$

Therefore, P(pivot | Vote) and P(pivot | Abstain) will be:
1) For player 1: P(pivot | Vote)= φ_1 and P(pivot | Abstain)= φ_3
2) For player 2: P(pivot | Vote)= φ_1 and P(pivot | Abstain)= φ_2
3) For player 3: P(pivot | Vote)= φ_2 + φ_3 + φ_4 and P(pivot | Abstain)= φ_6 + φ_7 + φ_8

Take φ_1= 0.7 , φ_4= 0.3 , $\varphi_i = 0$, $\forall$ i $\notin$ { 1 , 4 }

This solution satisfies the conditions for correlated equlibria:

For player 1:

$\varphi_1 \geq \frac{0.05}{0.3}(\varphi_1 + \varphi_2 + \varphi_5 + \varphi_6\ )$

$\varphi_3 \leq \frac{0.05}{0.3}(\varphi_3 + \varphi_4 + \varphi_7 + \varphi_8)$

For player 2:

$\varphi_1 \geq \frac{0.05}{0.3}(\varphi_1 + \varphi_3 + \varphi_5 + \varphi_7)$

$\varphi_2 \leq \frac{0.05}{0.3}(\varphi_2 + \varphi_4 + \varphi_6 + \varphi_8)$

For player 3:

$\varphi_2 + \varphi_3 + \varphi_4 \geq \frac{0.05}{0.4}(\varphi_1 + \varphi_2 + \varphi_3 + \varphi_4)$

$\varphi_6 + \varphi_7 + \varphi_8 \leq \frac{0.05}{0.4}(\varphi_5 + \varphi_6 + \varphi_7 + \varphi_8)$

The idea was to choose a proper positive probability for the case when dominant shareholder votes and everybody else abstains (φ_4 here). Since the right hand side of the first inequality for the dominant shareholder sums up to 1 , any left value greater than $\frac{\epsilon}{\theta_D}$ will satisfy this inequality. Of course, the value choosen should not be that large to destroy the inequalities for the other players. Indeed if we denote φ_1= 1 - $\alpha\ \epsilon$, φ_4= $\alpha\ \epsilon$, $1 \leq \alpha \leq \frac{1}{\epsilon}$-1

It is easy to see that this "correlated equilibria" is outside of the convex hull of the Nash equilibrium. Notice that no marginal distribution over individual strategies can induce a joint distribution over strategies of three players such that merely φ_1 and φ_4 are positive and the rest are strictly zero.

5 Existence of Correlated Equilibria

Theorem : If the dominant shareholder votes for change and if there exist a MWC such that $\forall j \in B$ such that $\Theta_j < \Theta_{MWC} - \Theta_D$ then there exist a correlated equilibria of the game which is not a subspace of the convex hull of Nash equilibrium.

Existence Proof Partition the outcome space into three subspaces. The first subspace denoted by S_D includes the outcome where only the dominant shareholder votes and any other player abstains. The second subspace denoted by S_{MWC} contains the outcome where only the dominant shareholder and exactly one MWC coalition participate and anybody else abstains. The third subspace includes all other outcomes.

As correlated equilibrium is a probability distribution over outcome space, one can assign three probabilities to these regions which should sum-up to one. Moreover, every correlated equilibrium should satisfy the individual rationality conditions (3) and (4). These help to characterize the solution space.

Call the probability assigned to S_{MWC} by φ_1 and that of S_D by φ_2 and the rest by φ_3. Denote the number of shares of the smallest participant in the MWC by $\underline{\Theta}$. Choose $\frac{\epsilon}{\underline{\Theta}} < \varphi_1 < 1 - \frac{\epsilon}{\Theta_D}$ and then let φ_2=1-φ_1 that implies φ_3=0. Any value of φ_1 and φ_2 in this region constitutes a correlated equilibrium.

To see the proof look at two rationality conditions (3) and (4) for the three classes for the players introduced in the previous paragraph:

1) **Dominant Shareholder :** We chose $\frac{\epsilon}{\Theta_D} < \varphi_2$ which satisfies (3). Moreover, the dominant shareholder never abstains therefore the left hand side of (4) is zero and the conditions holds automatically.

2) **Minimal Winning Coalition :** φ_1 was chosen in way such that $\frac{\epsilon}{\underline{\Theta}} < \varphi_1$ which satisfies equation (3) for MWC. The members of MWC can potentially be pivotal in many outcomes of the game, but all these outcomes come with zero probabilities. Moreover, they abstain at the outcome where the dominant shareholder is the only voter but are not pivotal at this circumstance. Therefore the left hand side of condition (4) is zero and it is fulfilled.

3) **Other Players :** For this type of players, condition (3) holds immediately since they do not votes at any outcome and therefore the right hand side of (3) is zero for them. But still, one may wonder if the condition (4) might be violated for some of these players. Under the assumptions stated at the beginning of the theorem, this condition also holds. To see this, first consider the player who belongs to A. Being pivotal for her means to change the outcome in favor of A at situations where the dominant shareholder wins the game. This ,however, is

impossible since we already know that no player can individually do that. Next, look the situation for the players belonging to B. To be pivotal they need to change the outcome S_{MWC} but this is not possible since the theorem is proposed only when there is no member of B who can join D and win MWC.

5.1 Correlated Equilibria Outside of Convex Hull of Nash Equilibrium

In the previous section an existing proof for some components of C.E was presented but one still needs to show that it contains equilibria outside of the convex hull of the Nash equilibrium. But before proceeding to the proof, I remind the reader of an intuitive lemma.

Lemma: Suppose S is a set of points in a n-dimensional space. Denote by H the convex hull of points in S. Moreover denote by h_m the m-th coordinate of any h∈H . Then for any h∈H and for any coordinate m , $min\{\, s_m : s \in S\} \leq h_m \leq max\{\, s_m : s \in S\}$. In words, it says that for any point in the convex hull of S, every coordinate of this point is bounded by the min and max of that coordinate among all members of S.

Proof: Every coordinate of any point in H is a convex combination of the coordinates of points in S. Therefore $h_m = \sum_{s \in S} \alpha_i s_m$, $0 \leq \alpha_i \leq 1$, $\forall m, M_{min} = min\{\, s_m\}, M_{max} = max\{\, s_m\}$, it is trivial that $M_{min} \leq \sum_s a_i * s_m \leq M_{max}$ therefore $M_{min} \leq h_m \leq M_{max}$. QED

Using this lemma I show that the suggested correlated equilibria indeed does not fall into the convex hull of Nash equilibrium.

Take any correlated equilibrium constructed as before remembering that $\frac{\epsilon}{\Theta} < \phi_1$ and recall the notations, N.E is the set of Nash equilibria, C.E the space of correlated equilibria and C the correlated equilibria component constructed in the previous section. Define Q: N.E → C.E. Q maps every Nash Equilibrium into its counterpart in the space of correlated equilibria. If σ_i be the probability of playing 1 for each player at certain Nash equilibrium, then the probability of any outcome S will be given by the product of these marginal probabilities. Function Q produces the probability distribution over all outcomes based on a particular Nash equilibrium and Q(N.E) means all correlated equilibria induced by Nash equilibria. The goal of this section is to show that

C.E $\not\subset$ C.H(Q(N.E))

Step 1: Denote by S_{MWC} the outcome at which MWC and D vote and other players abstain. In all correlated equilibria proposed before $\phi(S_{MWC}) \geq \frac{\epsilon}{\underline{\Theta}}$. By the way of negation, assume that all these correlated equilibria are covered by the convex hull of some Nash equilibria i.e. C$\subset$ C.H(Q(N.E)). By Lemma1 $C_1 = \phi(S_{MWC}) \geq$ max $Q_1(\sigma)$, $\forall$ $\sigma \in$ N.E

Hence $\exists\sigma \in N.E : \prod_{i\in MWC\cup\{D\}} \sigma_i \prod_{i\in I-MWC\cup\{D\}}(1-\sigma_i)$ $\geq C_1 > \frac{\epsilon}{\underline{\Theta}} \Rightarrow \sigma_i > \frac{\epsilon}{\underline{\Theta}}, \forall i \in MWC \cup \{D\}$ and $\sigma_i < \frac{\epsilon}{\underline{\Theta}}, \forall i \in I - MWC \cup \{D\}$

This means that to have C covered by the convex hull of *Q(N.E)* there should exist at least one Nash equilibrium where MWC and Dominant shareholder both play with "high" probabilities and everybody else plays with very "low" probability. High means $\sigma_i \geq \frac{\epsilon}{\underline{\Theta}}$ and low means $\sigma_i \leq \frac{\epsilon}{\underline{\Theta}}$. We show that this is impossible under the structure assumed for this game.

Step 2: Assume that C $\in C.H(Q(N'))$ and take any $N' \subset N.E$ which supports this point in C.E. Then from the lemma, $C_i = \sum_{\sigma\in N'} \lambda_k Q_i(\sigma), 0 \leq \lambda_k \leq 1$, $\exists$ j: $\lambda_j \neq 0$. On the other hand $C_{-1,-2}$=0 $\Rightarrow$ $Q_{-1,-2}(\sigma)$=0 , $\forall$ $\sigma \in$ N'. In other words, if there exist a subset N' of N.E which C lies in its convex hull then all elements of N' whose coefficient is non-zero should induce zero probabilities into all pure strategy combinations except S1 and S2 (otherwise C lies out of the convex hull since these coordinates of C are all zero). This immediately implies σ_i=0 , $\forall$ i $\in$ I-MWC $\cup$ D and $\forall$ $\sigma \in$ N' meaning that all players except MWC and D should play with pure strategy 0 in all of these Nash equilibria.

Step 3: Consider the probability of being pivotal $\forall$ i $\in$ MWC at Nash space. Every member of MWC might be pivotal in some outcomes including the case where σ_j=1 , $\forall$ j $\in MWC\cup$ $\{D\}$ and σ_j=0 , $\forall$ j $\in I - MWC\cup$ $\{D\}$. Denote this outcome by S'. Notice that $p(S') > \frac{\epsilon}{\underline{\Theta}}$. Now $\forall$ i $\in$ MWC look into P_i(pivot)=P(S')+P' , P' $\geq 0 \Rightarrow \sigma_i$=1 (if the probability of being pivotal is greater than $\frac{\epsilon}{\Theta_i}$ the player will play pure strategy 1). Now consider the probability of being pivotal for the dominant shareholder. Since we showed in the step 2 that σ_i=0 $\forall$ i $\in$ B-D and also showed in this step that σ_i=1 $\forall$ i $\in$ MWC , the dominant shareholder's probability of being pivotal is zero implying that

σ_D=0 but this contradicts the assumption that $\sigma_D > \frac{\epsilon}{\underline{\Theta}}$ Q.E.D

5.2 Concluding Remarks

A correlated equilibria component which differs from the Nash solution of a shareholder costly voting game was proposed. From an economic point of view this means that a carefully chosen coalition of shareholders can reach to some outcomes where the dominant shareholder is defeated there. Knowing this, a non-dominant shareholder who is contemplating between voting and staying at home, sees a positive expected return to her action (benefits of keeping the current policies running less the cost of voting) and therefore decides to vote with a strictly positive probability.

The implication of this result would be that the manager should not merely favor the dominant shareholder's preferences because in expectation there are other outcomes where she might be replaced by other group of shareholders. The proof is based on the concept of minimal winning coalitions and such coalition is not neccessarily unique. Thereofore the more the number of such subset is, the higher is the power of the party opposing the dominant shareholder.

Since the goal of this paper is to provide an existence result I did not try to characterize and demonstrate several other equilibria which exist even when some assumptions of the paper do not hold. One however should be careful not to go too far with the current results. This is mainly due to the existence of infinitely many correlated equilibria in this particular game. I just identified and studied one of these equilibria but the other equilibria may also exist which give other insights from the theory. There are several possible extensions for current model. One can look for other equilibria without restricting the attention to the minimal winning coalitions. Moreover, if the equilibria can be characterized in general, the value of each share can be determined based on the marginal contribution of the share to the winning coalitions.

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